ITERATIVE CODE-AIDED PHASE NOISE SYNCHRONIZATION
BASED ON THE LMMSE CRITERION

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ABSTRACT

In this paper, we examine code-aided synchronization in the presence of carrier frequency uncertainties and phase noise. As code-aided synchronization can only achieve high estimation accuracy if the initial parameter offset is sufficiently small, we employ a data-aided coarse synchronization unit comprising a feed-forward frequency estimator and a LMMSE estimator for pre-compensating frequency offsets and phase noise. Furthermore, we make use of known iterative synchronization techniques for frequency and phase offsets. We here extend the concept of iterative synchronization to iterative phase noise compensation and we show analytically under which circumstances our approach can achieve a worthwhile improvement in terms of estimation accuracy.

1. INTRODUCTION

With the invention of Turbo Codes and the rediscovery of Gallager’s LDPC codes, reliable transmission can be achieved at significantly reduced signal-to-noise ratios (SNR) as compared to before. However, when burst transmission is considered, synchronization is a major issue at these low SNRs [1].

With DVB–RCS [2], a standard for satellite uplink communication has been introduced whose implementation is very challenging due to the aforementioned issues: Transmission at low SNR coupled with short burst lengths. The satellite terminals are supposed to be both low-cost and low-power consuming. Therefore, in addition to large frequency offsets between transmitter and receiver that can occur, phase noise caused by the local oscillators is a major issue [3].

Usually, periodically transmitted pilot symbols are used for estimating the phase noise process. The LMMSE (least mean square error) algorithm optimally interpolates these observations in order to estimate the process for the positions of the unknown data symbols [4]. However, the achievable estimation accuracy strongly depends on the number of pilot symbols provided and on the SNR. In the considered case of DVB–RCS, this results in poor estimation accuracy and, therefore, poor bit error rate (BER) performance.

DVB–RCS foresees a parallel concatenated turbo code. The iterative processing and the high sensitivity of turbo codes against synchronization errors, immediately imply code-aided synchronization, often referred to as turbo synchronization [5]. As shown in various publications, e.g. [6] and [7], turbo synchronizers can achieve a performance in terms of estimation accuracy equivalent to that of data-aided (DA) estimation, even though (only) unknown data symbols are exploited. According to [5], turbo synchronization can be mathematically described by means of the Expectation Maximisation (EM) theory [8].

There already exist publications that apply the concept of turbo synchronization to the phase noise estimation problem. However, to our knowledge, these approaches, e.g. [9], [10], [11], base their algorithms on the assumptions of a random-walk (Wiener) model. Although it is shown, that this is a valid approximation, it is not strictly optimum in the sense of the LMMSE criterion when a specific power spectral density (PSD) is given to describe the phase noise process, as in [3]. Furthermore, as mathematical foundation they base their work on the Factor-Graph framework [12] and not on the EM theory.

In this paper, we extend the turbo synchronization approach that is based on the EM theory to phase noise estimation. We design our estimator optimum in the sense of the LMMSE criterion (Wiener filter) for the PSD given in [3]. We consider a whole system approach incorporating coarse synchronization, turbo synchronization, and decoding and we show that in the presence of phase noise, an iterative phase noise cancelation can significantly improve the system performance in terms of both, BER and convergence speed.

The paper is structured as follows: Section 2 and Section 3 point out the transmission system and the considered pilot symbol constellation. The derivation of the Wiener filter for phase noise synchronization is summarized in Section 4. In Section 5 and Section 6 the coarse synchronization unit and the turbo synchronization unit is treated. Simulation results are shown in Section 7.

2. TRANSMISSION MODEL

Information bits are grouped into packets of $L$ bits, are encoded with a turbo code and are mapped onto a QPSK mod-
ulation alphabet $A$. The resulting $K_d$ data symbols are then transmitted over an additive white Gaussian noise (AWGN) channel together with $K_p$ pilot symbols. $\eta$ denotes the ratio $\eta = K_p/(K_p + K_d)$. Let $K$ denote the entire set of time instants. The subsets $K_d$ and $K_p$ are related to data and pilot symbols, respectively.

Under the assumption of perfect symbol timing $^1$, the received baseband signal after sampling and matched filtering can be modeled as

$$r_k = a_k \cdot e^{j(2\pi T k + \theta_k)} + n_k,$$

with $T = 1/R_T$ being the symbol duration, and $R_T$ being the symbol rate. The transmitted and the received symbol at sampling instant $k$ are denoted as $a_k$ and $r_k$, respectively. Unit energy symbols are assumed, i.e. $E_s = E(|a_k|^2) = 1$. Furthermore, $n_k$ are the samples of complex-valued AWGN with independent real and imaginary parts, each having zero-mean and variance $\sigma_n^2 = N_0/(2E_s)$. In addition to the additive noise component, phase noise $\theta_k$ according to the specification [3] and a frequency offset $\Delta f$ are introduced by the physical channel and the local oscillators.

3. PILOT SCHEMES

Since DVB–RCS transmission takes place at very low SNR, pilot symbols for data-aided coarse synchronization are inevitable. For bandwidth efficiency, it is desirable to keep the ratio $\eta$ as low as possible.

DVB–RCS foresees a pure preamble of variable length. However, a preamble constellation is suboptimal for phase noise and frequency synchronization. The theoretically optimal constellation for frequency synchronization is splitting the pilot sequence into two equal parts, and placing one part as pure preamble at the beginning and one part as pure postamble at the end of the burst. Theoretically optimal for phase noise synchronization is distributing the pilot symbols across the burst equidistantly. Obviously, the optimal constellation for performing both frequency offset and phase noise synchronization is expected to be between these two extremes.

We here consider the two traffic bursts formats for DVB–RCS, the ATM burst ($L = 424$) and the MPEG2 burst ($L = 1504$). Phase noise estimation is more challenging for the longer bursts, whereas efficient frequency offset estimation is more problematic for the shorter burst. The latter is due to the fact that the performance of data-aided frequency estimators relies mainly on the amount of energy present in the pilot symbols, i.e. $K_p \cdot E_s/N_0$. Given a constant bandwidth efficiency $\eta$ for both bursts, $K_p$ is forcefully smaller for the ATM burst than for the MPEG2 burst, and such is $K_p \cdot E_s/N_0$. We, therefore, choose a spaced pre-/postamble structure for the ATM burst (the spacing is chosen to 6) and a pre-/mid-/postamble (P-M-P) structure for the MPEG2 burst.

$^1$Timing synchronization is handled on higher layers for DVB–RCS.

Fig. 1. Phase noise mask $\Phi_{\text{PN}}(f)$ for $R_T \geq 128$ kBaud [3], $\Phi_C(f)$ for $R_T = 128$ kBaud, $K = 2448$ after errorfree $\hat{\theta}$, $\Delta f$.

The P-M-P pilot constellation is dimensioned such that pre- and postamble consist of $K_p/4$ each. The residual $K_p/2$ pilot symbols are distributed equally between pre- and postamble. A more detailed motivation on the dimensioning of the spacing is given in [13].

It can be shown analytically [13] for these pilot constellations that for the ATM burst $\eta > 0.11$ has to be met in order to guarantee reliable coarse frequency synchronization, whereas $\eta > 0.08$ is sufficient for the MPEG2 burst.

4. DERIVATION OF THE WIENER FILTER FOR PHASE NOISE COMPENSATION

Phase noise estimation is done via a linear filter operation, where the filter coefficients can be generated offline based on the LMMSE-criterion [4]. The estimation process can be described as data-aided, as it solely relies on the observation of the known pilot symbols. As the pilot spacing is non-equitant (see Section 3), for simpler notation, we here introduce the vector $\mathbf{p}$ that contains all time indices $k \in K_p$, and a vector $\mathbf{d}$ that contains all time indices $k \in K_d$:

$$\mathbf{p} = [p_1, \ldots, p_{K_p}]^T, \quad \mathbf{d} = [d_1, \ldots, d_{K_d}]^T.$$ 

In accordance with [4], we consider planar filtering. The filter process can then be approximated as

$$\hat{\vartheta}_d = \arg \left\{ \mathbf{w}^T \mathbf{x} \right\},$$

with

$$\mathbf{w} = \mathbf{C}_{xx}^{-1} \mathbf{C}_{x\vartheta} \mathbf{d},$$

and $\mathbf{x}$ denoting the concatenation of the unmodulated symbols $x_k = r_k a_k^*$ with $k \in K_p$ and $(\cdot)^*$ denoting the complex conjugate.
conjugate. $C_{xx}$ is the covariance matrix given by:

$$C_{xx} = R_{\vartheta \vartheta} + I \sigma_n^2$$

$$= \begin{pmatrix} r_{\vartheta \vartheta}(0) + \sigma_n^2 & \cdots & r_{\vartheta \vartheta}(|p_1 - pK_p|) \\ \vdots & \ddots & \vdots \\ r_{\vartheta \vartheta}(|pK_p - p_1|) & \cdots & r_{\vartheta \vartheta}(0) + \sigma_n^2 \end{pmatrix}, \quad (4)$$

where the autocorrelation function $r_{\vartheta \vartheta}(t)$ can be obtained via the Fourier transform of the power spectral density $\Phi_{\text{PN}}(f)$ of the phase noise specified in [3] (see Fig. 1). Note that the total noise power is defined as $2 \sigma_n^2$. However, as we estimate the phase, the observation sequence $x$ is only affected by its tangential part, i.e. $\sigma_n^2$. The cross correlation vector $C_{x \vartheta}$ captures the influence of the different pilot symbol positions on the estimate at time index $d$ and is given as

$$C_{x \vartheta}^{[d]} = \begin{pmatrix} r_{\vartheta \vartheta}((d - p_1)) \\ \vdots \\ r_{\vartheta \vartheta}((d - pK_p)) \end{pmatrix}. \quad (5)$$

Based on (2), the mean squared error (MSE) for the phase estimate at each time index $d$ can be calculated as

$$E \left\{ (\vartheta_d - \hat{\vartheta}_d)^2 \right\} = r_{\vartheta \vartheta}(0) - C_{x \vartheta}^{[d]} \Phi_C C_{x \vartheta}^{[d]}. \quad (6)$$

The mean attainable MSE for all data symbol positions $d$ within the burst is denoted as:

$$\sigma_{\vartheta,WF}^2 = \frac{1}{K_d} \sum_{k \in K_d} E \left\{ (\vartheta_k - \hat{\vartheta}_k)^2 \right\}. \quad (7)$$

## 5. FEED-FORWARD SYNCHRONIZATION

The feed-forward (FF) synchronization unit consists of a frequency offset estimator and a Wiener filter described in the previous section to estimate the phase noise. If the Wiener filter is omitted, it is substituted by a simple phase offset estimator that only yields the average phase estimate for the whole burst. Note that due to the low operating SNR and threshold effects of (non-data-aided) estimators, the whole feed-forward synchronization unit relies solely on known pilot symbols and operates therefore data-aided.

For frequency offset estimation, we make use of a hierarchical synchronization chain, where each step consists of an estimation followed by a correction. The estimation is of increasing accuracy, but covers a smaller frequency range. Details concerning this estimation process are not the focus of this paper and can be found in [13]. However, it is important to note that the estimation accuracy achieved by this feed-forward estimator is optimum in terms that its MSE coincides with the Cramér-Rao Bound (CRB) that corresponds to amount and position of the known pilot symbols.

As frequency synchronization is performed prior to phase noise synchronization, the frequency estimator forcefully reduces linear components of the phase noise. Therefore, it changes the statistical properties of the phase noise process $\Phi_{\text{PN}}(f)$. If frequency and phase offset estimation was errorfree, $\Phi_{\text{PN}}(f)$ would degenerate to $\Phi_C(f)$, which is exemplarily shown in Fig. 1 for the MPEG2 burst coded at rate 1/3 and transmitted with symbol rate $R_t = 128$ kBaud. The frequency at which the left bend occurs is labeled $f_{cut}$ and occurs at $f_{cut} (0) = 1/T_{burst}$, where $T_{burst}$ denotes the duration of the burst $K/R_T$. This is due to the fact that frequency components smaller than $1/T_{burst}$ can be partially compensated for by a linear approximation. The right bend occurs at the cut-off frequency $f_{cut}$, which is according to Nyquist and matched filtering given by $f_{cut}(0) = R_t/2$.

We can easily see that the shorter the burst length, the more accurate the phase noise can be approximated linearly and the less power persists within the residual phase variation. Mathematical, we can express this relation by

$$\sigma_{\vartheta,FP}^2 = 2 \int_0^{R_t/2} \Phi_C(f, K, R_T) df, \quad (8)$$

where the subscript FP stands for errorfree Frequency and Phase offset synchronization. The residual standard deviations (STD) (see (8)) of the phase variation across the burst are given in Tab. 1.

### Table 1. Achievable STD by $\Delta f$, $\hat{\vartheta}$ for $R_T = 128$ kBaud.

<table>
<thead>
<tr>
<th>$L$</th>
<th>$\sigma_{\vartheta,FP}$[deg]</th>
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<tbody>
<tr>
<td>424</td>
<td>732</td>
</tr>
<tr>
<td>1504</td>
<td>2448</td>
</tr>
</tbody>
</table>

Comparing the values in Tab. 1 to the attainable MSE by data-aided Wiener filtering (see (7), first row in Tab. 2 and Tab. 3), it becomes obvious that $\sigma_{\vartheta,FP} < \sigma_{\vartheta,WF}$. However, at these low SNRs the linear components of the phase noise can of course not be estimated errorfree. Therefore, the results in Tab. 1 can only be viewed as a lower bound and phase noise synchronization by Wiener filtering is still reasonable.

Especially for the MPEG2 burst, the achievable MSE by purely data-aided Wiener filtering is rather high, leading to high performance losses in terms of convergence speed of the turbo decoder and in terms of the achievable BER. Supporting simulation results will be shown later in Section 7. Turbo synchronization as introduced by [5] is an approach to combat these effects, as will be shown in the following section.

## 6. TURBO SYNCHRONIZATION

The principle of turbo synchronization is to iteratively improve the estimation accuracy of the unknown parameter vector and the decoding result. In case of perfect convergence,
turbo synchronization achieves correct detection of all transmitted data symbols and can, thus, use them as pilot symbols. It is important to note that in this case, all symbols (data symbols and pilot symbols) are treated identically by the synchronization unit, as they then have equal reliability. For more details concerning the principle of turbo synchronization and the theoretical foundation - the Expectation Maximization (EM) theory - please refer to [5].

As proposed in [5], we carry out only one single decoding iteration prior to updating the frequency and the phase estimates. The optimum frequency estimator is prohibitively complex, as it requires an exhaustive search of the maximum of the likelihood function. We, therefore, employ the equally well performing estimator from [14].

When regarding iterative phase noise synchronization, we still rely on the use of so-called soft-symbols [5], that will be denoted \( \alpha_k \) in the sequel. The soft-symbols can be calculated from the a-posteriori outputs of the channel decoder (the rules are given in [5]). As already indicated, soft-symbols are equivalent to pilot symbols when the system converges. However, if the system is iterating, the amplitude of each soft-symbols scales with its reliability. The approach can then be called soft-decision directed (SDD). We, here, extend the principle of Wiener phase noise estimation to soft-symbols. In order to avoid high complexity at receiver side, we introduce a sliding window of length \( N_W \) that limits the length of the Wiener filter. It is symmetric, meaning that for generating an estimate at time index \( d \) the observation sequence \( x_{N_W} \) contains \((N_W - 1)/2\) elements before and after \( d \), i.e.:

\[
x_{N_W} = [x_{d-(N_W-1)/2}, \ldots, x_{d+(N_W-1)/2}]
\] (9)

with

\[
x_k = r_k \alpha_k.
\] (10)

Tab. 2 and Tab. 3 summarize the achievable MSE of an iterative Wiener filter approach. Note that the calculated MSE is based on (7) and is only valid for the case that the system converges. It can be seen that of course, the longer \( N_W \) is chosen, the more accurate the estimation of the phase noise.

The theoretical results shown in the tables already indicate two important results. Firstly, we see that it is not reasonable in terms of receiver complexity to choose the maximum filter length \( N_W \) as a saturation in terms of achievable accuracy is already achieved for smaller filter lengths and that the potential gains that could be possibly achieved by the maximum filter length \( N_W = K \) are rather negligible. Secondly, comparing Tab. 2 with Tab. 1, we note that even iterative phase noise synchronization cannot beat errorfree frequency and phase offset estimation. This result would be opposed to the LMMSE theory and is, of course, only valid for errorfree frequency and phase offset synchronization, that is not possible in reality and especially not at these low SNRs. However, it can be inferred from \( \sigma_{\alpha,WF} < \sigma_{\theta,WF} \) that turbo synchronization of the frequency and the phase offset will already approach the optimum quite well for the ATM burst \((L = 424)\).

## 7. SIMULATION RESULTS

The simulations in Fig. 2 show the frame error rate \((FER)\) performance versus the number of decoding iterations for the traffic bursts ATM \((L = 424)\) and MPEG2 \((L = 1504)\). As our focus is the phase noise synchronization, the initial frequency offset is set to \( \Delta f = 0 \) and the symbol rate is set to the worst case, i.e. \( R_T = 128 \text{ kBaud} \) [3].

The curves correspond to the receiver realization with only feed forward (FF) frequency and phase estimation (cross markers), with FF and turbo frequency and phase estimation (square markers), with FF Wiener phase noise estimation (diamond markers), with FF and turbo Wiener phase noise estimation for \( N_W = 101 \) (triangular markers) or \( N_W = 301/501 \) (pentagram markers) and without any synchronization error (dashed line).

As already hinted at in Section 6, Fig. 2(a) shows that iterative phase noise synchronization is not a reasonable receiver algorithm for the ATM burst. Turbo synchronization for compensating the linear components of the phase noise achieves similar performance with far less receiver complexity.

However, the results depicted in Fig. 2(b) indicate that iterative phase noise synchronization can save up to three decoding iterations when processing the MPEG2 burst and can, furthermore, lower the error floor. Results not presented here indicate that regarding the energy consumption in the transmitter, turbo synchronization enables a given frame error rate at reduced \( E_b/N_0 \).

<table>
<thead>
<tr>
<th>Table 2. Achievable STD by Wiener filtering; (L = 424, r = 1/3, E_b/N_0 = 1.6 \text{ dB}, R_T = 128 \text{kBaud.})</th>
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</thead>
<tbody>
<tr>
<td>( N_W )</td>
</tr>
<tr>
<td>DA Wiener filter</td>
</tr>
<tr>
<td>SDD Wiener filter</td>
</tr>
<tr>
<td>301</td>
</tr>
<tr>
<td>732</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 3. Achievable STD by Wiener filtering; (L = 1504, r = 1/3, E_b/N_0 = 1.2 \text{ dB}, R_T = 128 \text{kBaud.})</th>
</tr>
</thead>
<tbody>
<tr>
<td>( N_W )</td>
</tr>
<tr>
<td>DA Wiener filter</td>
</tr>
<tr>
<td>SDD Wiener filter</td>
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<tr>
<td>501</td>
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<tr>
<td>2448</td>
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Regarding the complexity, it can be said that the complexity of a code-aided phase noise estimation for one iteration is roughly of the same order as the complexity of a MAP decoder iteration.

8. CONCLUSION

In this paper, we consider synchronization issues for a system with short to medium range bursts and low transmit energy, i.e., DVB–RCS. After introducing a whole system approach, we examine to what extent iterative phase noise synchronization based on the Expectation Maximisation theory is a valuable approach to save transmit energy and improve the FER performance. It is shown that iterative phase noise cancelation can significantly improve the FER performance for medium length bursts. As the alternative would be to provide more pilot symbols, it also helps to improve the bandwidth efficiency.

9. REFERENCES


Fig. 2. FER vs. Iterations, Phase Noise [3], \( \Delta f = 0 \).